Explicit methods for stiff ODE with application to dynamics of swimming spirochetes

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New class of Runge-Kutta-Chebyshev methods allows significant increase of average time-step of explicit Euler method for stiff differential equations. We created several programs, which are based on RKC idea, such as RKC, DUMKA, ROCK2 and DUMKA3. The programs have been used for variety of problems, from Nuclear-Transport Equations to non-Newtonian flows, and dynamics of the elastic media
(see. http://www.math.tulane.edu/ amedovik/citationindex/citationindex.html). In particular, we used it to model mechanics of spirochete motility. We model spirochete as a 3D elastic mechanical system, but in order to simplify mechanical part of the model we represented it as a system with several rotational degrees of freedom. This system creates propulsive forces from two superhelicies rotating into opposite directions. The interaction of mechanical system and fluid are modeled via Stokes equations. Since the Stokes problem is linear we can use the Green tensor. Yielding, on surfaces of the swimming object, a relationship between intrinsic forces in the structure and velocities of the liquid.

$$u_j(\vec{x}_0) = -\frac{1}{8\pi\mu}\sum_{i-1}^{3}\int_{\partial D} G_{ij}(\vec{x},\vec{x}_0)(f_i^+(\vec{x}) - f_i^-(\vec{x}))dS(\vec{x}) \quad (1)$$

$$G_{ij}(\vec{x},\vec{x}_0) = \frac{\delta_{ij}}{r} + \frac{(x_i - x_{i,0})(x_j - x_{j,0})}{r^3} \quad (2)$$

As the result, we get system of ODEs. Where right hand side is a non-linear function of the geometry of the object. We used 3 dimensional B-spline to construct complex 3-dimentional geometries, and we were able to analyze static properties of the system. We also conducted dynamical simulations solving this nonlinear system, which described interaction of elastic body and viscose fluid by DUMKA3 solver. Simulations gave us enough confidence that we were able to understand mechanics of spirochete motility, and later experimental work conducted at NYU [3] with super-helical bodies, confirmed some for our conclusions. Excited by results of numerical simulations, and experiments of Professor Stuart Goldstein (Department of Genetics and Cell Biology at the University of Minnesota) conducted on living creatures. We started building mechanical robot which would simulate swimming spirochete. The robot is not yet finished, but some mechanical functionality will be presented. This work was conducted with several of my colleagues: Professors V.I.Lebedev, Assyr Abdulle, Ricardo Cortez, Lisa Fauci, Stuart Goldstein.

References

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